Optimal inflation targeting rule under positive hazard functions for price changes*

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March, 2015

Abstract

This paper reconsiders optimal inflation targeting in a model where persistence is generated by rational choices of the price-makers because of time-dependent pricing mechanism. In this framework, which generalizes the traditional Calvo model, inflation persistence is intrinsic, as it is micro-founded assuming that firms' pricing decisions depend on the time spent from the last price reset. We use a linear-quadratic approach to study the welfare effects and optimal policies. We disentangled two distortion sources showing how welfare falls in both the average of the probability of changing prices and its distribution among different firms. Described the underlying distortions of our setup, we analyze its normative implications for optimal inflation targeting. The issues of uncertainty and robustness are also considered, as we study robust optimal targeting policies. By using robust control techniques, we in fact consider the consequences of implementing “wrong” monetary rule due to a misinterpretation of sources of inflation inertia.


Keywords: time-dependent price adjustments, intrinsic inflation persistence, hybrid Phillips curve, optimal inflation targeting rule, robustness.

1 Introduction

Developments of New Keynesian theory show that modern central bankers should pursue a flexible-inflation targeting to implement optimal policy. Optimal flexible-inflation targeting consists of a commitment to a targeting rule that allows short-run departures of projected inflation from the long-run target rate (Woodford, 2003: Chapter 1). Optimal monetary policy

*The authors are grateful to Barbara Annicchiarico, Guido Ascarì, Efrem Castelnuovo, Bob Chirinko, Giuseppe Ferrero, Peter McAdam, Salvatore Nisticò, Ricardo Reis, Lorenza Rossi, Massimiliano Tancioni, Patrizio Tirelli, seminar participants at the Universities of Rome (Sapienza CMC conference), Crete, Manchester, Mannheim (ZEW conference), Tor Vergata, for useful comments on earlier drafts. The authors also acknowledge financial support by Sapienza University of Rome.
rules must deviate from inflation long-run target reacting to current and past deviations of other macroeconomic variables. This occurs because central bankers own just one instrument and during the business cycle they have multiple goals, i.e., they face trade-offs. For instance, in the standard New Keynesian model, after a cost push shock, central bankers need to take account of the inflation-output stabilization trade-off. Optimal inflation targeting rules have been extensively explored by different points of view (see, e.g., Svensson, 2003; Woodford, 2003; Svensson and Woodford, 2004; Giannoni and Woodford, 2005).

A very general feature of optimal targeting once forward-looking private-sector behavior is taken into account is that optimal policies are almost always history dependent. Optimal policy rule should react to the economy’s recent history of some target variables and not simply to the set of their possible state-contingent paths. The rationale is that by committing to a history dependent rule, central bankers can “manipulate” private sector expectations to improve the current trade-off they face. A commitment to respond later to past conditions can in fact shift expectations at the earlier date in a way that helps to achieve the central bank’s stabilization objectives. As stressed e.g. by Svensson and Woodford (2005), a forward-looking target criterion that involves only the projected paths of the target variables from the present time onward cannot determine an equilibrium with the optimal responses to disturbances. Instead, a history dependent target criterion is needed to implement optimal policies.

According to the above discussion, the general principle says that optimal target criterion should be history dependent; however, specific feature of the economy determines, e.g., how far in the future the inflation and output projections should extend and how much optimal rules should depend on recent conditions.

This paper reconsiders the issue of optimal inflation targeting in a model where persistence is generated by a time-dependent pricing model. By deriving the optimal inflation targeting rule in this environment, we aim to answer to policy questions such as what is the degree to which an optimal target criterion should be history dependent and what sort of additional variables ought to matter in a target criterion in a model with inflation intrinsic (or structural) inertia. In this framework, inflation persistence is obtained from the assumption that pricing decisions depend on the time spent from the last price reset. Moreover, differently from other popular models of inflation inertia, it is intrinsic and generated by rational behaviors (see Sheedy, 2007a; Yao, 2011). A welfare measure consistent with our framework and optimal targeting policy is obtained by using the linear-quadratic (LQ) approach proposed by Rotemberg and Woodford (1997) and further extended by, among others, Woodford (2002, 2003), Benigno and Woodford (2003, 2005, 2012). We compare our policy implications to those emerging from other models with inflation persistence. Moreover, we use our measure to analyze the peculiar distortions stemming from intrinsic inflation persistence generated by a vintage-dependent pricing mechanism.

We borrow from Sheedy (2007a, 2010) a log-linear economy representation which departs from the standard Calvo mechanism assuming that a firm has a higher probability of changing its price the longer it has kept it unchanged. This price assumption leads to a hybrid time-dependent Phillips curve exhibiting intrinsic inertia à la Fuhrer and Moore (1995), as it
embeds a backward-term for inflation. Formally, our pricing model implies a non-negative hazard function. It generalizes the Calvo price setting, which emerges as a particular case when the hazard is flat. Evidence for positive sloped hazard function is provided by, e.g., Sheedy (2010) and Di Bartolomeo and Di Pietro (2015). Instead, micro evidence on price setting about the slope of the hazard function is mixed. Results depend on sample, countries, periods considered and methodologies used. However, a discussion about the empirical debate on the evidence of positive hazard functions in micro-economic data is outside the scope of the present paper. We refer to Sheedy (2007a, 2010), Yao (2011, 2014) or Nakamura and Steinsson (2013) for more details.

Our model presents a richer pricing mechanism, implying the existence of different distortions compared to the standard Calvo model. In the Calvo price setting, where the probability of resetting a price is randomly assigned, all firms have the same probability of changing their prices independently of their history, i.e., the time elapsed since their last update. Distortions are only related to one feature of the price setting mechanism, i.e., the (average) probability of resetting a price, which is equal to the probability of individual firms. Instead, in our environment, distortions can be associated to different average probabilities, as in Calvo, but also to different distributions of the probability of resetting price, since this may be not the same among firms. In other words, our setup generalizes Calvo (1983) and allows us to disentangle the distortions associated to different average from those stemming from different distributions of the probability of resetting prices.

We use our welfare approximation to explore the nature of macro distortions induced by our vintage-dependent pricing model. Our approximation, in fact, makes explicit how welfare costs are generated by time-dependent pricing adjustments. These costs are distinct from those associated with relative price dispersion and consumption fluctuations that appear in the standard New Keynesian model.

As said, we follow Woodford’s (2003) approach. We assume the presence of an output or employment subsidy that offsets the distortions due to the market power of monopolistically-competitive price-setters, so that the steady state under a zero-inflation policy involves an efficient level of output. We consider an approximation around an efficient steady state to compare our results to those of Steinsson (2003) and Giannoni and Woodford (2005), who investigate alternative models with inflation persistence.

Steinsson (2003) considers optimal policies in Galí and Gertler (1999) setup, where a fraction of the producers in the economy set their prices according to a rule-of-thumb, thus

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1Di Bartolomeo and Di Pietro (2015) also check the stability of the slope with respect to policy regime changes. They conclude that the positive hazard slope is regime invariant. See Benati (2008, 2009) for a general discussion on the point.
2Among others, Cecchetti (1986), Göette et al. (2005) and Ikeda and Nishioka (2007) find strong support for increasing hazard functions. By contrast, Alvarez et al. (2005) do not; Nakamura and Steinsson (2008) find that it is initially downward sloping and then flat.
3In particular, the technical problems related to the hazard estimations from micro data are discussed by Nakamura and Steinsson (2013), who emphasize that “the failure of the empirical methods to fully purge the effects of heterogeneity is a potential concern regarding evidence of downward-sloping hazard functions.”
4The approach can be however generalized to the case of a distorted steady state (Benigno and Woodford, 2005, 2012).
generating structural inflation persistence.\(^5\) The Phillips curve is a convex combination of a forward- and a backward-looking inflation term. As long as the relative importance of the backward term relatively to the forward one increases, he finds that slumps (booms) are more persistent; commitment to optimal inflation targeting rule implies more gradual policies; losses under optimal policies grow. By contrast, the relative gains of commitment over discretion fall in the degree of inflation inertia. Thus, the relative advantage of manipulating expectations that characterize the commitment decreases.

Giannoni and Woodford (2005) also investigate optimal inflation targeting in a New Keynesian model with inflation persistence. Following Christiano et al. (2005), they introduce inflation inertia by assuming that individual prices are indexed to past inflation during the intervals between re-optimizations of the individual prices and use a consistent utility-based welfare measure derived as in Woodford (2003). Giannoni and Woodford (2005) derive a closed form for the inflation targeting rule and show that optimal deviations from the long-run inflation target should also depend on past inflation. They find that as the degree of inflation inertia increases, optimal policies imply higher variability in inflation. However, the optimal path of the output gap is unaffected by inertia as well as the optimal loss. The rationale of their results is that past inflation enters in the model economy and in the welfare function in the same manner. Therefore, optimal policy is not different from the standard one by accounting of a quasi-differenced inflation rate, i.e., an inflation target that also depends on past inflation. As a result, changes in the quasi-differenced inflation rate mimic changes of the inflation target in standard model with no inflation inertia one-by-one.

Another paper closely related to our work is Sheedy (2007b), who considers a very general setup, where no specific assumptions about price stickiness are imposed and attempts to derive some general principles about optimal policies, i.e., some robust insights. The aim of Sheedy (2007b) is to look at optimal policy under a general perspective; the case of positive hazard functions for price changes with no differences across sector is used as an example. This example is further discussed by Woodford (2011: 792-798).\(^6\) Our point of view is the opposite; and it somehow complements their works. In fact, we only focus on positive hazard functions for price changes, investigating the nature of the distortions stemming from this assumption and its implications for optimal targeting policy. Specifically, regarding optimal policies, we differ from him since we investigate the welfare properties of the optimal inflation targeting and the impact on it of different degrees of structural persistence. Furthermore, we consider the relative gains of optimal inflation targeting over alternative policy regimes.\(^7\)

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\(^5\)See also Amato and Laubach (2003).

\(^6\)Differently from Woodford (2011), we derive a specification for the welfare loss that is more similar to the standard one and therefore it is more useful for comparisons. The loss derived by Michael Woodford in fact depends on the optimal price dynamics and thus requires an additional equation in the model. However, Woodford (2011) performs neither a welfare analysis nor regime comparisons.

\(^7\)It is worth noticing that, although Sheedy (2007b) does not provide welfare comparison under different regimes, he derives alternative policy rules. Among these, he computes a sort of discretionary optimal policy. This rule, labeled “Markovian,” is obtained by ignoring the effects of the policy on lagged terms (see example 2 in his paper). We instead compare the effects of optimal inflation targeting to the traditional discretionary policy obtained by the solution of the Bellman equation as in Walsh (2003: Section 8.4.5). The two discretionary policies do not lead to the same results (results are available upon request).
As concerning with more fundamental sources of model uncertainty and the robustness of monetary policy strategies to uncertainty are generally viewed as important (see e.g. McCallum, 1988; Taylor, 1993; Levin, 2003; Hansen and Sargent, 2011), we also study the consequences of implementing “wrong” targeting rules due to a misinterpretation of sources of inflation persistence by assuming a considerable amount of uncertainty concerning the correct specification of the model representing the economy. Inspired by Sbordone (2007), we look at the robustness of these targeting policies by using robust control techniques.

The rest of the paper is organized as follows. Section 2 presents the economic setup considered and explains how to embed a New Keynesian Phillips curve with intrinsic persistence by using positive hazard functions; it shows the derivation of the welfare function when inflation inertia is achieved via a reset price probability that is time-dependent and explores the nature of macro distortions induced by our pricing model. Section 3 studies the implication of inflation inertia for the conduct of optimal targeting policies. We analyze how a change in the hazard slope affects the welfare gain of a commitment over discretion. Furthermore, we consider the cost for the Government of misunderstanding the sources of persistence in implementing its optimal policy. A final section concludes.

2 Positive hazard functions, welfare and distortions

2.1 The price setting mechanism

The core of the New Keynesian approach is price stickiness; i.e., a situation in which the nominal price is resistant to change. The most common way to introduce it is the Calvo staggered contracts model, where each firm faces a sort of lottery for resetting price. In every period of time, each firm has a constant probability of being able to change its price. The alternative mechanism we consider here is instead a lottery where the probability of resetting the price, defined by a hazard function, is a function of time. In line of principle a hazard function can be upward or downward sloping. According to the empirical evidence found in Sheedy (2007a, 2010), we assume that a firm has a higher probability of updating its price the longer it has kept it unchanged (i.e., positive slope). Now, the firm faces a probability distribution over possible probabilities of resetting conditional to the time elapsed from the last adjustment.

Formally, in order to derive the Phillips curve, we first need to specify the hazard function, which is defined as the event rate at time $h$ conditional on survival until time $h$ or later, where the event is “be able to reset the price.” The hazard function can be written as:

$$\alpha (h) = \alpha_p + \sum_{j=1}^{\min(h-1,n)} \varphi_p (j) \left\{ \prod_{k=h-j}^{h-1} [1 - \alpha (k)] \right\}^{-1},$$

where $\alpha (h)$ is the probability to change a price which last reset was $h$ periods ago; $\alpha_p$ is the

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initial level of the hazard, $\varphi_p(j)$ is its slope; $n$ is the number of parameters that control the slope, for $n = 1$ the slope is governed by only one parameter, $\varphi_p(j) = \varphi_p$.

Knowing (1), the representative firm $i$ chooses its price to maximize the profit, as it enjoys monopoly power on the goods market, constrained by the total demand for its product (i.e., $Y_t(i) = (P_t(i)/P_t)^{-\varepsilon_p} Y_t$, where $P_t(i)/P_t$ is the relative price, $Y_t$ is the aggregate demand, and $\varepsilon_p$ is the firm’s price elasticity). We assume that a representative firm has the following production function: $Y_t(i) = A_t N_t(i)^{1-\delta}$, where labor ($N_t$) is the only factor of production, $A_t$ denotes the technology, and $\delta \in (0, 1)$. Formally, the representative firm $i$ solves

$$\max_{P_t(i)} \sum_{t=0}^{\infty} \varsigma_t E_0 \beta^t \left\{ P_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t - W_t \left[ \frac{1}{A_t} \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t \right]^{\frac{1}{1-\delta}} \right\}$$

where $\varsigma_t = \prod_{h=1}^{l} (1 - \alpha_h)$ with $\varsigma_0 = 1$ represents the probability that a price remains fixed for $l$ periods (survival function) and $\beta$ is the discount factor.

By assuming a competitive labor market, under very common assumptions in New Keynesian framework, the solution of the problem (2) leads to the following log-linear inflation equation:

$$\pi_t = \Psi_1 \pi_{t-1} + \beta \Psi_2 E_t \pi_{t+1} - \beta^2 \Psi_1 E_t \pi_{t+2} + \kappa x_t + u_t$$

where $\pi_t$ is the inflation rate; $x_t$ is the output gap; $u_t$ is an i.i.d. cost push shock. If not differently stated, we indicate deviations from the steady state with lowercase letters. Coefficients in (3) are:

$$\left\{ \begin{array}{c} \Psi_1 = \left( 1 - \alpha_p \right) - \varphi_p \left[ 1 - \beta \left( 1 - \alpha_p \right) \right] \\
\Psi_2 = 1 + (1 - \beta) \Psi_1 \\
\kappa = \left( \alpha_p + \varphi_p \right) \left[ 1 - \beta \left( 1 - \alpha_p \right) \right] \left[ 1 - \varphi_p \left[ 1 - \beta \left( 1 - \alpha_p \right) \right] \right]^{-1} \Xi_p \\
\end{array} \right.$$

with $\Xi_p = \left( \sigma + \tfrac{\gamma + \delta}{1 - \delta} \right) \tfrac{1 - \delta}{1 - \delta + \varphi_p}$, where $\sigma$, $\gamma$ are parameters characterizing the representative consumer’s utility function (see (4), below). It is worth noticing that a steeper hazard, given by higher $\varphi_p$, entails a more persistent component of the Phillips curve as $\Psi_1$ increases.

Equation (3) is our Phillips curve. Coefficient $\kappa$ measures its slope and also depends on the hazard function parameters ($\alpha_p$, $\varphi_p$). This Phillips curve captures the intrinsic component of inflation inertia, since it has a backward term. The negative coefficient attached to $\pi_{t+2}$ must be interpreted as a reduction in the total degree of its forwardness. Equation (3) nests the standard New Keynesian Phillips curve as a special case for $\varphi_p = 0$ (flat hazard): under this case, $\alpha_p$ represents the expected probability of price adjustment.

The underlying intuition of (3) can be explained as follows. Observing a temporary shock to the real marginal cost, optimizing firms at $t_0$ will raise their relative prices. As a result the general level of prices will also rise. Then at $t_1$, optimizing firms can be of two kinds

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9The reader is referred to Sheedy (2007a) for the complete derivation. For the sake of simplicity, we assume $n = 1$. An alternative, but equivalent, derivation is described in Sheedy (2010).

10Although they will not fully exploit the mark-up over the marginal cost since they expect that tomorrow the shock will vanish, but they do not known if they will be able to reset the price again.
(selection effect): those who have changed their price at $t_0$ and those who did not. The former need to reduce their relative prices (roll-back), the latter to increase (catch-up)—since now the shock is vanished, but average relative prices are higher and nominal quantities should be aligned to the new general level of prices. In the Calvo’s setup the two opposite effects cancel out. Under a positive hazard function, it is more likely to extract in the Calvo’s lottery a firm who has not adjusted its price than one who did it. Therefore, the catch-up effect tends to prevail over the roll-back, creating inflation inertia as inflation tends to continue to be positive. This explains the lagged term in (3). A symmetric argument explains the negative coefficient associated with $E_t \pi_{t+2}$. The inflation persistence implied by (3) is thus structural since we observe it, but the fundamentals explaining inflation themselves exhibit no persistence.

2.2 A consistent welfare measure

The welfare loss derives from a second-order Taylor approximation of the representative household’s utility function and mainly depends on the form that takes the price dispersion. The details of the approach developed by Rotemberg and Woodford (1997, 1999) are widely discussed in Woodford (2003: Chapter 6), Galí (2008: Chapter 4), Benigno and Woodford (2012). In the derivation we follow Galí (2008: Chapter 4), from whom we only differ in the price dispersion evolution, which in our case relies on the use of time-dependent price setting with non-constant hazard functions.

We assume a utility function taking the following separable form:

$$U_t = \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right)$$

(4)

where $C_t$ is consumption; $\sigma$ and $\gamma$ are parameters.

A second-order approximation of (4) is:

$$U_t - U \simeq U_c Y \left( y_t + \frac{1-\sigma}{2} y_t^2 \right) + U_N N \left( n_t + \frac{1+\gamma}{2} n_t^2 \right) + t.i.p. + O \left( \| \xi^3 \| \right),$$

(5)

where $\sigma = -\frac{U_c}{U_N} Y$, $\gamma = \frac{U_c}{U_N} N$; $U$, $Y$, $N$ are steady state values and $t.i.p.$ denotes the terms independent of policy. Note that to obtain (5), we have used the aggregate resource constraint $C_t = Y_t$.

By integrating the production function, $Y_t = \int_0^1 Y_t(i) di = A_t \left[ \int_0^1 N_t(i) di \right]^{1-\delta} = A_t N_t^{1-\delta}$, we can manipulate the demand function of good $i$ to obtain:

$$\left( \frac{Y_t}{A_t} \right) \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\gamma}{1-\delta}} di = N_t$$

(6)

where $D_t$ measures the degree of price dispersion.
Log-linearizing (6) around the steady state implies:

\[(1 - \delta) n_t = y_t - a_t + \frac{\xi_p}{2\Theta} \text{var}_i \{p_t(i)\} \tag{7}\]

since up to a second-order approximation \((1 - \delta) d_t \simeq \frac{\xi_p}{2\Theta} \text{var}_i \{p_t(i)\}\), where \(\Theta = \frac{1 - \delta}{1 - \delta + \delta \xi_p}\) and \(p_t(i) = \log(P_t(i))\); \(\text{var}_i \{p_t(i)\}\) is the cross-sectional variance of prices.

We consider an efficient steady state, thus \(-U_n/U_c\) equals the marginal product of labor in the steady state, given by \((1 - \delta) Y/N\). Next, by substituting (7) into (5), after some simple algebraic manipulations, we obtain:

\[
\frac{U_t - U}{U_c Y} \simeq -\frac{1}{2} \left[\frac{\xi_p}{\Theta} \text{var}_i \{p_t(i)\} + \left(\sigma + \frac{\gamma + \delta}{1 - \delta}\right) x_t^2\right] + t.i.p. + \mathcal{O}(\|\xi^3\|), \tag{8}\]

where \(x_t = y_t - y_t^n\) denotes the output gap and \(y_t^n\) is the natural level of output. Then, as usual, we can express our welfare function as follows:

\[
W = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\xi_p}{\Theta} \text{var}_i \{p_t(i)\} + \left(\sigma + \frac{\gamma + \delta}{1 - \delta}\right) x_t^2\right] + t.i.p. + \mathcal{O}(\|\xi^3\|) \tag{9}\]

In order to specify our welfare criterion, we need to find an expression that relates \(\text{var}_i \{p_t(i)\}\) to \(\pi_t\). In our framework, as shown by Sheedy (2007a), the aggregate price level evolves as:

\[
\log P_t = \sum_{h=0}^{\infty} \theta_h \log P_{t-h}^r \tag{10}\]

where \(P_{t-h}^r\) stands for the reset price and \(\theta_h\) denotes the share of firms using a price which last change was \(h\) periods ago. Thus, price level is an average of past reset prices weighted by the share of firms using such price at time \(t\).

By making use of (10) and exploiting the properties of the variance, we can show that the discounted sum of price dispersion evolves in the following way:

\[
\sum_{t=0}^{\infty} \beta^t \Delta_t = \sum_{t=0}^{\infty} \beta^t \left[\frac{(\pi_t - \varphi_p \pi_{t-1})^2}{\alpha_p + \varphi_p} - \pi_t^2\right] \frac{1}{1 - \beta (1 - \alpha_p - \varphi_p)} + \mathcal{O}(\|\xi^3\|), \tag{11}\]

the proof is illustrated in Appendix.

We substitute (11) into (9) and obtain our welfare measure:

\[
W = -\Omega \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \Gamma_1 x_t^2 + \Gamma_2 \pi_{t-1}^2 + \Gamma_3 \pi_t \pi_{t-1}\right] + t.i.p. + \mathcal{O}(\|\xi^3\|) \tag{12}\]
where \( \Omega, \Gamma_1, \Gamma_2, \) and \( \Gamma_3 \) are expressed as follows:

\[
\begin{align*}
\Omega &= \frac{\varepsilon_p}{\Theta \left[ 1 - \beta \left( 1 - \alpha_p - \varphi_p \right) \right] \left( \alpha_p + \varphi_p \right)}, \\
\Gamma_1 &= \frac{1}{\Omega} \left( \sigma + \gamma + \delta \right), \\
\Gamma_2 &= \frac{\varphi_p^2}{1 - \alpha_p - \varphi_p}, \\
\Gamma_3 &= -2 \frac{\varphi_p}{1 - \alpha_p - \varphi_p}.
\end{align*}
\]

(13) \hspace{1cm} (14) \hspace{1cm} (15) \hspace{1cm} (16)

As for the Phillips curve, our welfare measure (12) encompasses that deriving from Calvo price setting for \( \varphi_p = 0 \): in such a case the weight attached to backward inflation drops to zero.

### 2.3 Averages and distributions of reset probability and distortions

In the standard Calvo pricing model, the source of distortions is the (average) probability of resetting prices, which is constant among firms in each instant of time. Distortions and price duration are in fact mapped one-by-one to this probability. In other words, firms are ex ante homogenous, facing the same probability of being extracted in the Calvo’s lottery. In our generalization of Calvo (1983), instead, we can disentangle two sources of distortions stemming from the average and the distribution of the probability of resetting prices. Here reset probability is not constant, unless the hazard is flat. Firms are ex ante heterogeneous, as each face a different probability of resetting according to the time elapsed since the last reset.\(^{11}\) Distortions are related to the average probability of resetting prices (as in Calvo standard model), but now they also depend on how the reset probability is distributed among firms.

A given average probability of resetting prices (ARP) can be obtained for different slopes of the hazard function (\( \varphi_p \)).\(^{12}\) For instance, we can obtain three different scenarios consistent with \( ARP = 0.3 \) associated to three different distributions of the reset probability across firms, as illustrated by the figure below.

\(^{11}\)Of course, two firms who have reset at the same time have the same probability to re-adjust.

\(^{12}\)By using the following relationship: \( ARP = \alpha_p + \varphi_p \).
All the panels of Figure 1 imply $ARP = 0.3$, but Panel (a) is built by assuming $\varphi_p = 0$ (Calvo price model); in Panel (b) $\varphi_p = 0.1$; in Panel (c) $\varphi_p = 0.25$.

The figure shows that as long the hazard function becomes steeper, there is more cross-sectional dispersion that implies low probability of reset for firms who have recently adjusted their prices and high probabilities for the firms who have set their prices in the far past. It is clear that in Panel (c) there is a greater dispersion in the probability to adjust a price, i.e., there is heterogeneity in the reset probability across firms. Similar figures can be drawn for different average probabilities.

Given the average reset probability, we can refer to the scenario (a) as Calvo; scenario (b) as the mid-dispersion case; (c) as the high-dispersion case. It is worth noting that given the $ARP$ level, Panels (a) shows that the probability of being able to post a new price is constant and equal among all the firms in the Calvo scenario. By contrast, if dispersion in the reset probability is not zero, given the $ARP$ level, more dispersion implies lower individual probability for firms that have recently changed their prices and higher probability for those that did it in the far past to compensate and keep the $APR$ constant.

Table 1 reports the welfare effects associated to several average probabilities with different dispersions. For the sake of comparison, we consider the same monetary policy rule in all cases. We express the welfare effects in the more common form of welfare loss, which are further normalized to the Calvo scenario with a reset probability equal to 0.4 (i.e., our baseline). The table is built on a standard calibration of model parameters. We set the discount factor ($\beta$) equal to 0.99. The labor share is 2/3 so we set $\delta = 0.33$. We assume a net price mark-up of 20% setting $\varepsilon_p = 6$; we consider a log-linear utility function calibrating $\sigma = 1$ and $\gamma = 2$. The process governing the cost push shock is a white noise. Our qualitative results are independent of the calibration.

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13The table is built by considering a transitory cost push shock. Thus it reports conditional welfare.

14We consider a policy consistent to a Taylor rule, which only responds to current inflation by a coefficient equal to 1.5. The same results hold if optimal policies are instead considered. More details are available upon request.
Table 1 – Welfare loss for different price setting schemes

<table>
<thead>
<tr>
<th>Average reset probability (ARP)</th>
<th>Hazard function slope ($\varphi_p$)</th>
<th>0</th>
<th>0.1</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>1.000</td>
<td>1.230</td>
<td>1.647</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>1.348</td>
<td>1.685</td>
<td>2.338</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>1.509</td>
<td>1.897</td>
<td>2.686</td>
<td></td>
</tr>
</tbody>
</table>

The table shows that a decrease in the ARP induces more distortions as in the traditional setup. Moreover, here distortions also increase in the dispersion of the reset probability.

It is well known, in the Calvo’s setup, that the welfare losses associated to the lower values for ARP are related to the price dispersion. In a flexible price environment, the optimal pricing rule is always to apply a mark-up over the marginal cost. However, when price are sticky, firms do not know when they will be able to re-optimize, they thus must balance the cost to apply a mark-up over the current marginal cost against the benefit of staying close to the profit maximizing mark-up over time following their inflation expectations about the future and reset probabilities. This balance is costly and induces distortions, because implies an ex post heterogeneity and a difference in the relative prices that induces misallocation of resources. Distortions vanish in the zero-inflation long run when all relative prices are equals.

Positive hazard functions imply a sort of price race that enhances the distortions due to price dispersion associated with stickiness described above. By definition, everything equal (i.e. the ARP), positive hazard functions lead to more cross-sectional dispersion in the reset probability. As long as probability of firms who have recently adjusted falls, the probability of the other firms must rise. A period after a temporary positive shock to the real marginal cost, firms extracted from the Calvo’s lottery adjust their relative price—some will lower their prices due to the roll-back; others will increase them due to the catch-up. If the roll-back and catch-up do not compensate each other the general level of price will move and some firms will be forced to follow it. Specifically, average price will increase. In the next period, roll-backers need to reduce less their price but catch-up firms need to rise it more because the increase in the price level. Since the latter are more likely to be extracted, relative prices will take more time to be equalized at the long-run value; therefore, welfare costs are magnified.

The next subsection investigates how these two features of the price setting process affect the conduct of monetary policy under different assumptions about the policy regime (discretion and commitment), and how they affects the relative gains of the latter on the former.

3 Optimal inflation targeting and intrinsic inertia

3.1 Optimal targeting rule

The general Government’s problem consists in maximizing (12) subject to (3), conditional to the policy regime. Formally, the Government’s problem at some point of time $t$ can be
expressed as minimization of the following Lagrangian expression:

\[
\min_{\{\pi_{t+j}\}_{j=0}^{\infty}, \{x_{t+j}\}_{j=0}^{\infty}} E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{2} \left[ \pi_{t+j}^2 + \Gamma_1 x_{t+j}^2 + \Gamma_2 \pi_{t+j}^2 \right] + \lambda_{t+j} \left[ \pi_{t+j} - \Psi_1 \pi_{t+j-1} - \beta \Psi_2 \pi_{t+j+1} + \beta^2 \Psi_1 \pi_{t+j+2} - \kappa x_{t+j} - u_{t+j} \right] \right\}
\]

where \( \lambda_{t+j} \) is the Lagrangian multiplier. Corresponding first order conditions are:

\[
(1 + \Gamma_2 \beta) \pi_{t+j} + \frac{\Gamma_3}{2} \pi_{t+j-1} + \beta \frac{\Gamma_3}{2} E_{t+j} \pi_{t+j+1} + \lambda_{t+j} - \Psi_1 \beta E_{t+j} \lambda_{t+j+1} = 0 \quad \text{for } j = 0 \tag{18}
\]

\[
(1 + \Gamma_2 \beta) \pi_{t+j} + \frac{\Gamma_3}{2} \pi_{t+j-1} + \beta \frac{\Gamma_3}{2} E_{t+j} \pi_{t+j+1} + \lambda_{t+j} - \Psi_1 \beta E_{t+j} \lambda_{t+j+1} + \Psi_2 \lambda_{t+j-1} - \Psi_2 \lambda_{t+j-1} = 0 \quad \text{for } j = 1 \tag{19}
\]

\[
(1 + \Gamma_2 \beta) \pi_{t+j} + \frac{\Gamma_3}{2} \pi_{t+j-1} + \beta \frac{\Gamma_3}{2} E_{t+j} \pi_{t+j+1} + \lambda_{t+j} - \Psi_1 \beta E_{t+j} \lambda_{t+j+1} + \Psi_2 \lambda_{t+j-1} + \Psi_1 \lambda_{t+j-2} = 0 \quad \text{for } j > 1 \tag{20}
\]

\[
\Gamma_1 x_{t+j} - \kappa \lambda_{t+j} = 0 \quad \text{for } j \geq 0 \tag{21}
\]

As it is well known, the system (18)-(21) leads to a dynamic inconsistency. At time \( t \) the policymaker implements (18) and commits to act following (19) in \( t+1 \). When \( t+1 \) comes, it would be optimal to pursue again (18) rather than (19). Thus, time-inconsistency arises as at \( t+1 \) it is no longer optimal act as planned at \( t \).

Woodford (2003) proposes to overcome dynamic inconsistency by implementing an alternative regime known as “timeless perspective,” where the Government should ignore (18) and (19). The idea is that optimal policy solves problem (17) “from some date forward as being optimal from a timeless perspective, rather than from the perspective of the particular time at which the policy is actually chosen” (Woodford, 2011: 744). In such a case, the solution is given by equations (20) and (21).

At a generic time \( t \), combining (20) and (21), we eliminate \( \lambda_t \) and obtain the following optimal flexible targeting rule:

\[
\pi_t = -\frac{\Gamma_1}{\kappa(1 + \Gamma_2 \beta)} \left[ x_t - \Psi_1 \beta E_t x_{t+1} - \Psi_2 x_{t-1} + \Psi_1 x_{t-2} + \frac{\Gamma_3}{2} \frac{\kappa}{\Gamma_1} (\pi_{t-1} + \beta E_t \pi_{t+1}) \right] \tag{22}
\]

Optimal policy imposes to deviate from the long-run inflation target by considering past values of output and inflation and expected inflation.

---

15 The same dynamic inconsistency explained above arises also between \( t+1 \) and \( t+2 \).
As usual, optimal targeting under timeless perspective is characterized by the fact that the Government, by reacting to the past output gap, affects current inflation expectations and obtains a more favorable trade-off between inflation and output at the cost of a longer stabilization (solving the so-called stabilization bias problem). Compared to the standard textbook case, our Phillips curve (3) implies a trade-off between inflation and output gap that also depends on lagged inflation ($\pi_{t-1}$) and an additional forward term ($E_t \pi_{t+2}$). Thus, in order to manipulate expectations, the Government now finds optimal to react also to the expected future output gap ($E_t x_{t+1}$) and its past value for an additional lag ($x_{t-2}$). Moreover, our optimal inflation targeting rule calls for responding also to both a backward and an expected value of inflation. If $\varphi_p = 0$, $\psi_1$ drops to zero and the textbook case is obtained—as both lagged inflation and the additional forward term no longer affect the Phillips curve trade-off.

In order to implement optimal policy, as argued by Svensson and Woodford (2005), a history dependent target criterion is necessary. In fact, no target criterion which involves only the projected paths of the target variables from the present time onward is able to determine an equilibrium with the optimal responses to disturbances.

### 3.2 The gain of commitment

We study the effects of optimal targeting by numerical methods. Our simulation is performed by using the calibration reported in Section 2.3.

We illustrate the optimal targeting policy in Figure 2, which plots the impulse response functions to a one-standard-deviation cost push innovation in the high-dispersion scenario, assuming an average reset probability equal to 0.3.\textsuperscript{16} We plot the dynamic responses for both the optimal targeting rule (22) and a discretionary policy (non-inertial rule).

\textsuperscript{16}Simulations are performed by using the Söderlind’s (1999) toolkit.
It is clear from Figure 2 that when an optimal inflation targeting rule is implemented (solid line) the variability of both inflation and output gap is reduced with respect to the case in which monetary policy is conducted according to a non-inertial policy rule (dotted line): this entails smaller welfare losses. Under an optimal targeting rule, inflation initially jumps and then undershoots converging progressively to its steady state. Looking at the output gap equilibrium path, it remains below zero for a time and exhibits a gradual convergence to its long-run value. What we observe is that an optimal targeting policy involves a history dependent behavior: the optimal rule should consider both current and past values of the target variables (in our case inflation and output gap).

In Figure 3 we consider the impulse response functions associated to optimal inflation targeting for different degrees of inertia. Specifically, we consider the three scenarios discussed in Section 2.3 (i.e., Calvo, mid-dispersion and high-dispersion under a common average reset.
probability equal to 0.3).

The path followed by inflation and output gap is the same in each scenario. They only differ on a quantitative point of view. A greater dispersion in the reset probability leads to a larger variability in both inflation and output gap, and, as a consequence, higher losses. As inflation becomes more persistent, the deflation experienced by the economy is harder and involves a larger fall in the output gap that returns to its steady state more slowly. In other words, in presence of a greater dispersion in the reset probability is more difficult to stabilize the economy. The intuition of the result is twofold. As explained, this is due to the existence of inflation persistence, which makes a disinflation more costly. Moreover, in our setup, dispersion in the reset probability also leads to additional stabilization costs, since manipulation of expectations becomes more difficult when dispersion increases because of the presence of two expectations terms in the Phillips curve (with opposite signs).

Following Steinsson (2003), it is interesting to compare the relative performance of an optimal inflation targeting with respect to a non-inertial discretionary rule for different degree of inertia. Table 2 shows the relative gains of an optimal commitment on discretion—computed according to the utility-based welfare maximization. We consider different slopes for the hazard function starting from Calvo (\( \varphi_p = 0 \)) to the high-dispersion scenario (\( \varphi_p = 0.3 \)). In all cases we keep fixed the average probability of changing prices (\( APR \)) at 0.3. It is worth remembering that the inflation intrinsic persistence grows with \( \varphi_p \). Thus, the table reports the percentage gains of an optimal inflation targeting for different degree of inflation inertia.

<table>
<thead>
<tr>
<th>hazard function slope (( \varphi_p ))</th>
<th>0</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>% gain from commitment</td>
<td>31.2</td>
<td>34.4</td>
<td>38.0</td>
<td>42.0</td>
<td>46.5</td>
<td>51.5</td>
<td>55.8</td>
</tr>
</tbody>
</table>
The greater $\varphi_p$ is, the higher the relative gain of commitment is. This result differs from Steinsson (2003), who finds that the gain falls in the degree of inflation persistence. Indeed, he finds some increasing gains for low degrees of inertia. Gains of commitment are mainly related to the government ability to manipulate expectations in order to improve the inflation-output gap trade-off.

In Steinsson (2003) inflation inertia is introduced by a simple rule-of-thumb, and results come out from the fact that as the backward component of the Phillips curve increases, the forward-looking one falls, reducing the gain from commitment. By contrast, in our model, inflation inertia is driven by the slope of the hazard function and grows with the dispersion of the reset probability among firms. Optimizing firms know that, in the period immediately after their resets, they will be very unlikely to change their prices again (see Figure 1, Panel (c)). Thus, in choosing a new price, each firm gives much importance to the future state of the economy, when it will probably be allowed to reset again. As a consequence, commitment gains are higher in a world with high-dispersion as expectations on the future state of the economy matter more. Thus, here, implementing a policy rule able to influence private sector expectations may lead to an improvement of the inflation-output gap trade-off.

The relative effects of inflation inertia on the optimal policy regimes are thus not general but they depend on the way intrinsic inflation persistence is introduced.

### 3.3 Targeting and misinterpretation of persistence sources

Now we assume model uncertainty and consider the cost of misunderstanding the true sources of inflation persistence. In the spirit of Sbordone (2007), we compare the dynamics of implementing a stabilization policy when the policymaker overestimates the degree of intrinsic persistence to those arising from ignoring actual structural persistence. At the end of the section, by using robust control techniques (specifically, the Savage’s minimax regret criterion), we look at robust policies.\(^{17}\)

We consider two specifications, in both we set $ARP = 0.3$. The first specification is a purely forward-looking New Keynesian Phillips curve ($\varphi_p = 0$), in which persistence is extrinsic, i.e., it depends on the auto-correlation of the cost push shock. The second is the high-dispersion scenario ($\varphi_p = 0.25$) that endogenously generates (intrinsic) inflation persistence by way of a lagged inflation term in the New Keynesian Phillips curve.

In each specification we compute the two welfare losses associated to a conduct for monetary policy where the government attributes the observed persistence to an extrinsic or intrinsic nature. Of course, for each specification, only one policy rule is “correct,” while the other is based on a misinterpretation of the source of inflation persistence. Formally, the two rules are

\[
\pi_t = -\frac{\Gamma_1}{\kappa} (x_t - \Psi_2 x_{t-1})
\]  

\[
\pi_t = -\frac{\Gamma_1}{\kappa(1 + \Gamma_2 \beta)} \left[ x_t - \Psi_1 \beta E_t x_{t+1} - \Psi_2 x_{t-1} + \Psi_1 x_{t-2} + \frac{\kappa \Gamma_3}{2 \Gamma_1} (\pi_{t-1} + \beta E_t \pi_{t+1}) \right]
\]  

\(^{17}\)For a review of the issue see Hansen and Sargent (2011) and references therein.
where (23) is the optimal commitment for the standard Calvo case (extrinsic persistence, i.e., $\varphi_p = 0$) and (24) denotes the optimal inflation targeting rule followed when inertia is intrinsic, i.e., $\varphi_p = 0.25$. Formally, we want to investigate if there are welfare losses in acting following (24) when inflation is characterized by intrinsic inertia and we repeat the same exercise implementing (23) in a context where inertia is driven by the serial correlation of the shock (i.e., it is extrinsic).

Figure 4 plots the impulse response functions. It depicts the equilibrium paths of inflation and output gap to a cost push shock when the policymaker conducts a stabilization policy that is optimal conditional to his beliefs, which are not necessary correct. In Panel (a) we assume that the central bank implements rule (23) whatever the persistence source is. Similarly, Panel (b) considers the case of rule (24).

![Figure 4 - Inflation and output gap IRFs to a cost push shock under model uncertainty.](image-url)
The policy responses to a cost push shock are qualitatively similar independently of the true source of persistence or the rule implemented. Inflation shows an initial increase, offset by a mild deflation in the following quarters. This is the typical pattern of optimal inflation targeting.

Quantitatively, there is an evident difference depending on which is the correct source of inertia. Intrinsic inertia emphasizes the deflationary policy associated with the commitment. Instead, differences between the relative performances of the right and the wrong rule are less evident. However, they can be simply illustrated by looking at the initial response to the shock in terms of inflation. In the case of intrinsic inertia, the wrong rule implies a loose response in terms of inflation; optimal response in fact calls for slightly stronger inflation stabilization.\textsuperscript{18} The contrary occurs for the case of extrinsic inertia. Therefore, in both cases, the positive effects of the management of expectations on the trade-off between current inflation and output are not fully exploited. Depending on the case, when the wrong rule is implemented, expectations will result too high or too low compared to their optimal values. The quantitative effects of the inefficient management to expectations are illustrated in Table 3.

Losses in the two specifications under different policy rule are presented in following table. In the last line, we also compute the percentage loss deriving from the implementation of an optimal stabilization policy under wrong policymaker beliefs about the nature of inertia.

<table>
<thead>
<tr>
<th>Targeting policy:</th>
<th>True source of inflation persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>extrinsic ($\varphi_p = 0$)</td>
</tr>
<tr>
<td>Rule (23)</td>
<td>1.000</td>
</tr>
<tr>
<td>Rule (24)</td>
<td>1.003</td>
</tr>
<tr>
<td>Loss difference (%)</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Central bankers’ misunderstandings are always costly. Conducting a stabilization policy under wrong belief about the true source of inertia always entails welfare worsening due to an inefficient management of expectations.\textsuperscript{20} Table 3 illustrates how misunderstanding the true source of persistence implies an additional loss in both specifications. In per cent terms, the additional losses are about 0.3% when the true model is a purely forward-looking Phillips (i.e., extrinsic inertia) and 0.2% when inflation persistence is intrinsic. Ignoring intrinsic inflation inertia leads to a smaller additional loss.

\textsuperscript{18} This can be understood by comparing the inflation path in the two panels. Specifically, it means to compare solid lines in Panel (a) and (b). By contrast, if the source of persistence is extrinsic, we need to look at the dotted lines.

\textsuperscript{19} Losses are normalized to the case of correct policy without inflation structural persistence (where the welfare loss is the smallest.) An alternative consists of expressing all of them in consumption equivalent terms. The loss difference measures the additional cost of implementing the wrong policy.

\textsuperscript{20} It is worth noting that the result is not trivial since optimal timeless rules can be dominated by other inertial rules. See Blake (2001) or Jensen and McCallum (2002) for a discussion.
Both misunderstandings lead to some costs, so in a context of model uncertainty it is interesting to consider policy robustness. In a framework of our kind, as emphasized by Brock et al. (2007) and Sbordone (2007), a good robustness criterion is to select among different policy rules by using the minimax regret criterion developed by Savage (1951). The Savage's criterion can be defined by introducing the concept of regret. Let consider a policymaker facing uncertainty about the model describing the economy. For the sake of simplicity, as in our case, we consider two possible specifications for the economy (summarized in the set $M = \{m_A, m_B\}$) and two possible policies (summarized in the set $P = \{p_A, p_B\}$). The regret is the difference of the loss incurred by implementing certain policy in a certain given specification and the loss obtained by using the optimal policy in that given specification. Formally, for $p_i \in P$ and $m_j \in M$, the regret is $R(p_i, m_j) = L(p_i, m_j) - \min_{p \in P} L(p, m_j)$, where $L(p, m)$ is the loss. The Savage's robust policy $p^*$ is then obtained by applying the minimax to the regret, i.e. $p^* = \min_{p \in P} \max_{m \in M} R(p, m)$.

Robust policy is derived from Table 4, where we report the regret for the two different rules (policies) in our two alternative specifications of the economy. The robust policy is then chosen by applying the Savage's criterion.

<table>
<thead>
<tr>
<th>Targeting policy:</th>
<th>True source of inflation persistence</th>
<th>Rule (23)</th>
<th>Rule (24)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>extrinsic ($\varphi_p = 0$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>intrinsic ($\varphi_p = 0.25$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rule (23)</td>
<td></td>
<td>0</td>
<td>0.0029</td>
</tr>
<tr>
<td>Rule (24)</td>
<td></td>
<td>0.0029</td>
<td>0</td>
</tr>
</tbody>
</table>

The table confirms the results found in Table 3. Making a mistake about the origin of inflation inertia always involves incremental costs: a regret of 0.0029 arises when persistence is entirely generated by a serially correlated shock, whereas the regret is 0.0025 when the persistence is due to a backward component in the Phillips curve. The regret is minimized by the rule (23). Thus, under model uncertainty, the policymaker should always conduct monetary policy by implementing a targeting rule derived ignoring intrinsic inertia.

The intuition of our result is as follows. Now the central bank faces uncertainty on the effects of targeting on the economy since the true model is unknown. If the true source of persistence is extrinsic, but the central bank assumes that it is intrinsic, too loose policies compared to the optimal one will be implemented. By contrast, when the central bank wrongly assumes that the true source of persistence is structural, monetary policy is too aggressive to inflation. In a LQ context of model uncertainty, the effects of “wrong” policies are not symmetric: too aggressive policies are more costly; hence, robust policy actions should be more cautious (Brainard, 1967). In our case, when the central bank misunderstands the true model of the economy, the overreaction in the case of intrinsic persistence costs more that the under-reaction in the case of extrinsic persistence. As a result, if persistence is driven by structural factors, the costs of a wrong policy are higher. Thus under model uncertainty, robust policy implies to ignore extrinsic persistence.

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21See Stoye (2011) for its axiomatization.

22Effects can be modified in more complicated models.
4 Conclusions

In this paper, we reconsider optimal inflation targeting in a model with intrinsic inflation persistence by using the Rotemberg and Woodford’s (1997) linear-quadratic approach. Inflation inertia was micro-founded assuming that the probability to update a price for each firm is a function of the time; specifically, in our model firms’ pricing decisions depend on the time spent from the last price reset. By our welfare approximation, we explored the nature of macro distortions induced by our pricing model and its implications for the optimal monetary policy.

By generalizing the standard Calvo’s price mechanism, we disentangled two sources of distortions by considering both the average (as in Calvo) and the cross-distribution across firms of the probability of resetting prices. We showed that, in addition to the welfare costs due to higher average, further welfare losses are generated by increases in the dispersion of the probability of resetting prices.

After computing optimal inflation targeting policy, we found that the effects of inflation inertia on the optimal policy are different from those stemming from alternative models of inflation persistence based on rule-of-thumb assumptions or indexation mechanisms. In our setup the relative gain of commitment over discretion increases in the degree of the inflation persistence as—once intrinsic inflation is introduced by positive hazard function—the interaction between the different components of the Phillips curve becomes complex. Therefore, relative effects of inflation inertia on the optimal policy regimes are at least not general, but they depend on the way intrinsic inflation persistence is introduced.

Assuming model uncertainty, we finally analyzed the effects on optimal monetary policy of misinterpreting the sources of inflation inertia, which can be either intrinsic (driven by a backward component of the Phillips curve) or extrinsic (driven by the shock auto-correlation). Implementing the wrong rule is always costly whatever the right model is. Moreover, using robust control techniques, we found that robust policy requires to ignore intrinsic persistence and to implement a simple textbook timeless perspective policy. The result comes from the general Brainard’s cautious principle.

Appendix

The welfare function can be derived in three steps. We can write the cross-sectional mean and variance as 

\[ \bar{P}_t \equiv E_i p_t(i) \] 

and 

\[ \Delta_t^p = \text{var}_i [p_t(i)] \], 

where we define 

\[ p_t(i) = \log P_t(i) \].

1. As shown by Sheedy (2007a), the aggregate price level evolves according to (10) and thus the cross-sectional mean can be written as

\[ \bar{P}_t = E_i [p_t(i)] = (1 - \alpha_p) [E_i p_{t-1}(i)] - \varphi_p [E_i p_{t-2}(i)] + (\alpha_p + \varphi_p) p_t^* \quad (25) \] 

It follows that

\[ \bar{P}_t - \bar{P}_{t-1} = (1 - \alpha_p) [E_i p_t(i) - \bar{P}_{t-1}] - \varphi_p [E_i p_{t-2}(i) - \bar{P}_{t-1}] + (\alpha_p + \varphi_p) [p_t^* - \bar{P}_{t-1}] \]
from which we obtain
\[ p_t^* - \bar{P}_{t-1} = \frac{\pi_t - \varphi_p \pi_{t-1}}{\alpha_p + \varphi_p} \] (26)

2. The cross-sectional variance is
\[ \Delta_t = \text{var} \left[ p_t(i) - \bar{P}_{t-1} \right] = E_i(\left[p_t(i) - \bar{P}_{t-1}\right]^2) - (E_i[\left[p_t(i) - \bar{P}_{t-1}\right])^2 = E_i(\left[p_t(i) - \bar{P}_{t-1}\right]^2) - \pi_t^2 \] (27)

where, by (25), the first r.h.s. term is
\[ E_i(\left[p_t(i) - \bar{P}_{t-1}\right]^2) = (1 - \alpha_p) E_i(\left[p_{t-1}(i) - \bar{P}_{t-1}\right]^2) - \varphi_p E_i(\left[p_{t-2}(i) - \bar{P}_{t-1}\right]^2) + (\alpha_p + \varphi_p) \left[p_t^* - \bar{P}_{t-1}\right]^2 \] (28)

By using \( \Delta_{t-1} = E_i(\left[p_{t-1}(i) - \bar{P}_{t-1}\right]^2) \), \( \Delta_{t-2} = E_i(\left[p_{t-2}(i) - \bar{P}_{t-1}\right]^2) \) and plugging (26) and (28) in (27), we obtain
\[ \Delta_t = (1 - \alpha_p) \Delta_{t-1} - \varphi_p \Delta_{t-2} + \frac{(\pi_t - \varphi_p \pi_{t-1})^2}{\alpha_p + \varphi_p} - \pi_t^2 + O(\|\xi^3\|) \] (29)

3. Then integrating (29), the degree of price dispersion at any point of time from some initial period on is given by
\[ \Delta_t = (1 - \alpha_p - \varphi_p)^{t+1} \Delta_1 + \sum_{s=0}^{t} (1 - \alpha_p - \varphi_p)^{t-s} \left[ \frac{(\pi_s - \varphi_p \pi_{s-1})^2}{\alpha_p + \varphi_p} - \pi_s^2 \right] + O(\|\xi^3\|) \] (30)

By discounting, over all periods \( t \geq 0 \), we get (11), i.e.
\[ \sum_{t=0}^{\infty} \beta^t \Delta_t = \sum_{t=0}^{\infty} \beta^t \left( \sum_{s=0}^{t} (1 - \alpha_p - \varphi_p)^{t-s} \left[ \frac{(\pi_s - \varphi_p \pi_{s-1})^2}{\alpha_p + \varphi_p} - \pi_s^2 \right] \right) + O(\|\xi^3\|) \]
\[ = \frac{1}{1 - \beta (1 - \alpha_p - \varphi_p)} \sum_{t=0}^{\infty} \beta^t \left[ \frac{(\pi_t - \varphi_p \pi_{t-1})^2}{\alpha_p + \varphi_p} - \pi_t^2 \right] + O(\|\xi^3\|) \] (31)

Finally, placing (31) into (9) gives as an expression for the welfare
\[ W = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\varepsilon_p (\alpha_p + \varphi_p)^{-1}}{\Theta} \left( \pi_t - \varphi_p \pi_{t-1}\right)^2 - \pi_t^2 \right] + \left( \sigma + \gamma + \delta \right) x_t^2 ] + t.i.p. + O(\|\xi^3\|) \] (32)
References


